

# Exotic Magnetic Properties in $^{17}\text{C}$

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Magnetic dipole transitions in  $^{17}\text{C}$  are investigated by shell model calculations. The important role of the tensor interaction for magnetic dipole transitions in this exotic neutron-rich nucleus is pointed out. The recently observed anomalous quenching of the magnetic dipole transition in  $1/2_1^+ \rightarrow 3/2_{\text{g.s.}}^+$  is shown to be well explained by using a modified shell model Hamiltonian that takes full account of the tensor force and monopole corrections in the isospin  $T = 1$  channel. The predicted quadrupole moment of  $^{17}\text{C}$  is smaller than the value obtained by conventional shell model Hamiltonians.

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**Introduction.** Due to considerable advances in radioactive beam facilities, much progress has been made in the studies of exotic nuclei. Measurements of electric quadrupole ( $E2$ ) and low-energy dipole ( $E1$ ) transition strengths, as well as their energies, are powerful tools to study the change of shell structure toward drip-lines. The magicity at  $N = 8$  and  $N = 20$  has been found to vanish near the drip-lines [1]. Retardation of Gamow-Teller (GT) transitions in light neutron-rich nuclei,  $^{11}\text{Li}$  and  $^{12}\text{Be}$ , also supports the vanishing of  $N = 8$  magicity [2].

Recently,  $E2$  transitions in neutron-rich Be and C isotopes have been investigated. The  $E2$  transition strength observed in the  $0_2^+ \rightarrow 2_1^+$  transition in  $^{12}\text{Be}$  suggests large admixtures of the  $p$ -shell and  $sd$ -shell components in the wave functions of these states [3]. This large mixing between the shells is reproduced in a shell model calculation by using the SFO Hamiltonian [4].

Anomalous hindrance of  $E2$  transition rates in  $^{16}\text{C}$  and  $^{18}\text{C}$ , compared to the strengths in stable nuclei, has been found [5,6]. Effective charges which depend on excess neutron numbers are necessary to explain the observed hindrance. These phenomena suggest the decoupling of proton and neutron motions in exotic neutron-rich nuclei. On the other hand, a quantitative theoretical basis for this dependence remains an open question, requiring more advances in microscopic understanding of effective charges.

Studies of odd C isotopes have also been performed recently. Low energy excited states in  $^{17}\text{C}$  and  $^{19}\text{C}$  were measured by  $\gamma$  transitions [7,8]. Magnetic dipole transitions in  $^{17}\text{C}$  have been measured, and an anomalous quenching of the strength in the  $1/2_1^+ \rightarrow 3/2_{\text{g.s.}}^+$  transition has been found [8]. The rate of the  $5/2_1^+ \rightarrow 3/2_{\text{g.s.}}^+$  transition, on the other hand, was found to be normal. Here, we study the  $M1$  transitions in  $^{17}\text{C}$  by shell model calculations with the use of the SFO Hamiltonian and its modified version, taking into account the

important role of the tensor force consistent with the  $\pi + \rho$  meson-exchange interaction.

The tensor interaction is in general important for shell evolutions in nuclei. We have already revised the shell model Hamiltonian for the  $p$ -shell, and obtained considerable improvements in the descriptions of magnetic properties of  $p$ -shell nuclei [4]. In the present paper, we pay attention to the interaction between the  $p$ -shell and the  $sd$ -shell, and replace the tensor component of the previous interaction (SFO) by the tensor interaction arising from  $\pi + \rho$  meson-exchange. Note that the magnitude of the tensor force in the SFO Hamiltonian [4] is small compared to the case of the meson-exchange model. We shall also discuss the magnitude of the tensor interaction in other conventional Hamiltonians, for example, the Warburton-Brown interaction [9], in particular its role in the study of exotic carbon isotopes. We will show the dependence of the shell gap between proton  $0p_{1/2}$  and  $0p_{3/2}$  orbits on the mass number of the isotopes. Magnetic properties of  $^{17}\text{C}$  are shown to be sensitive to the tensor component of the interaction, and the anomalous behavior of the  $M1$  transitions will be found to be well described by using the tensor force of the  $\pi + \rho$  meson-exchange model. In addition to the tensor component, we also revised the  $T = 1$  monopole terms. This revision leads to the correct ordering of low-lying energy levels in  $^{17}\text{C}$  as well as to a reduction of the  $M1$  transition strengths. We will show that the quenching of the strength in the  $1/2_1^+ \rightarrow 3/2_{\text{g.s.}}^+$  transition is well described by our modified Hamiltonian.

**Modification of SFO Hamiltonian.** One of general properties of the tensor interaction, that is, the dependence of its monopole component on the spin-orbit coupling of the single particle states, was discussed in Ref. [10]. It has been pointed out that the monopole component of the  $\pi + \rho$  meson-exchange tensor force is attractive between  $j_> = \ell + 1/2$  and  $j_< = \ell - 1/2$  orbits, and repulsive between  $j_>$  and  $j_>$ , or  $j_<$  and  $j_<$ , orbits. This general rule is important in the shell evolution. The change of the shell structure and magic numbers toward drip-lines is induced by the tensor interaction, as shown

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by the behaviors of effective single particle energies in  $N = 8$  isotones [11] and Sb isotopes [10].

The SFO Hamiltonian has been obtained in Ref. [4] by improving the PSDMK2 Hamiltonian [12,13], whose  $p$ -shell part is the Cohen-Kurath Hamiltonian, (8–16)2BE [14], and the  $p$ - $sd$  crossing part is taken from the Millener-Kurath Hamiltonian [12]. The  $sd$ -shell part and the matrix elements  $\langle p^2 | V | (sd)^2 \rangle$  are from Kuo's  $G$ -matrix calculation [15,16]. The strength of the  $0p_{1/2}$ - $0p_{3/2}$  spin-flip matrix elements in the  $T = 0$  channel, and the gap between the  $0p_{1/2}$  and  $0p_{3/2}$  single particle energies have been modified as explained in Ref. [4].

As the tensor interaction affects magnetic properties directly, we first improve the SFO Hamiltonian further so that its tensor component becomes consistent with that of the  $\pi + \rho$  meson-exchange potential. Although the signs of the monopole terms are the same for the SFO and the meson-exchange interaction, their magnitudes in the SFO case are about one-third of the  $\pi + \rho$  meson-exchange model. We therefore replace the tensor component of the SFO interaction by the tensor component of the meson-exchange model, which effectively means to replace the tensor components of two-body matrix elements of the type  $\langle p(sd); JT | V | p(sd); JT \rangle$  by those of the  $\pi + \rho$  meson-exchange model. Moreover, the spin-orbit component of the interaction, which comes from  $\sigma + \omega + \rho$  meson exchanges, is replaced by that of the M3Y interaction [17] for the same type of two-body matrix elements. The spin-orbit components of the triplet odd matrix elements for  $1s_{1/2}$ - $0p_{1/2}$  orbits then become enhanced. Except for additional monopole corrections in the  $T = 1$  channel, which will be discussed later, the other two-body matrix elements, including the  $p$ -shell part, are unchanged, since the level structure and magnetic properties of stable  $p$ -shell nuclei are quite well described by the SFO Hamiltonian. The modified Hamiltonian will be referred to as SFO-tls hereafter.

In order to show the effects of the enlargement of the tensor and spin-orbit components of the interaction, we compare how the gap between the proton  $0p_{1/2}$  and  $0p_{3/2}$  shells is affected by filling the neutron  $0p_{1/2}$ ,  $1s_{1/2}$ , and  $0d_{5/2}$  orbits. The dependence of  $\Delta E_p$ , which is the difference between proton effective single particle energies for  $0p_{1/2}$  and  $0p_{3/2}$  orbits, on mass number  $A$  is shown in Fig. 1 for carbon isotopes. In the case of the solid lines, it is assumed that two neutrons are in the  $1s_{1/2}$  orbit for  $^{16}\text{C}$ , and for  $^{18}\text{C}$  the two additional neutrons are in the  $0d_{5/2}$  orbit. In the case of the dashed lines, one neutron is in the  $1s_{1/2}$  orbit and another in the  $0d_{5/2}$  orbit for  $^{16}\text{C}$ , while for  $^{18}\text{C}$  one additional neutron is in the  $1s_{1/2}$  orbit and three additional neutrons are in the  $0d_{5/2}$  orbit. As the tensor force pushes up (down) the proton  $0p_{1/2}$  ( $0p_{3/2}$ ) orbit when the neutron  $0p_{1/2}$  orbit is filled,  $\Delta E_p$  is enhanced at  $A = 14$  in both SFO-tls and SFO cases. This is not the case for WBP and WBT [9], where  $\Delta E_p$  decreases. Thus, in  $^{14}\text{C}$  the proton  $0p_{3/2}$  shell can be considered to be more inert for SFO-tls and SFO. When neutrons are added in the  $1s_{1/2}$  orbit, the shell gap further increases, especially for SFO-tls, where repulsive spin-orbit interaction between  $1s_{1/2}$  and  $0p_{1/2}$  shells is properly taken into account. With more neutrons in the  $0d_{5/2}$  orbit, the shell gap becomes smaller because the tensor force pulls down (up)

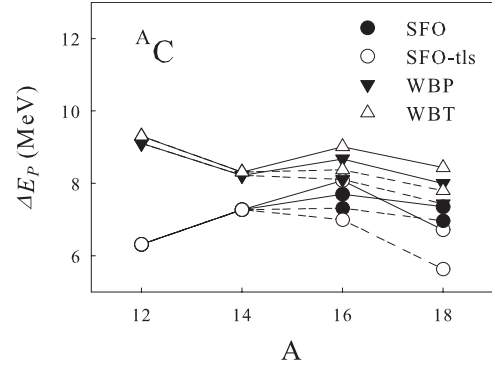


FIG. 1. The change of the shell gap of the proton  $0p$  orbits,  $\Delta E_p$ , that is, the difference between the effective single particle energies of the  $0p_{1/2}$  and  $0p_{3/2}$  shells for carbon isotopes with mass number  $A$ , for the SFO, SFO-tls, WBP, and WBT Hamiltonians. Solid lines denote the case where two  $1s_{1/2}$  neutrons are added after  $A = 14$ , and then two  $0d_{5/2}$  neutrons are added after  $A = 16$ . The dashed lines refer to the case where one  $1s_{1/2}$  neutron and then another  $0d_{5/2}$  neutron is added after  $A = 14$ , and two  $0d_{5/2}$  neutrons are added after  $A = 16$ .

the proton  $0p_{1/2}$  ( $0p_{3/2}$ ) orbit. The amount of the reduction is conspicuous for SFO-tls. The dependence of  $\Delta E_p$  on  $A$  is large for SFO-tls compared with other Hamiltonians. The shell gap becomes a maximum at  $A = 16$  or  $A = 14$  for SFO-tls and SFO, in contrast to WBP or WBT where the shell gap changes more slowly.

If the motion of valence neutrons is confined within the  $p$ -shell, the difference between the present results and those of Ref. [4] is small. On the other hand, if valence neutrons are excited from the  $p$ -shell to the  $sd$ -shell by a sizable amount, the difference becomes noticeable and one needs to use SFO-tls. For the former case, examples are found in low-lying energy levels, magnetic moments and GT transitions. Examples of notable differences are found in excitation energies of intruder states and  $B(E2)$  value for the  $0_2^+ \rightarrow 2_1^+$  transition in  $^{12}\text{Be}$ . The  $B(E2)$  value is calculated to be  $6.8(10.2) e^2 \text{fm}^4$  with the use of effective charges  $e_p = 1.3(1.5)$  and  $e_n = 0.4(0.5)$ , which is much closer to the recently observed value,  $B(E2) = 7.0 \pm 0.6 e^2 \text{fm}^4$  [3], than the result obtained in Ref. [4].

Now we study magnetic dipole transitions in  $^{17}\text{C}$  with the use of the SFO-tls. The shell model configuration space including up to  $(2-3)\hbar\omega$  excitations is used here, which is the same as in Ref. [4]. Corrections in the monopole terms are important to reproduce the ordering of energy levels of low-lying states in  $^{17}\text{C}$  as well as the  $M1$  transition strengths.

**Magnetic dipole transitions in  $^{17}\text{C}$ .** Recently, transition rates from low-lying excited states to the ground state ( $3/2_{\text{g.s.}}^+$ ) of  $^{17}\text{C}$  were measured. The strength for the  $1/2_1^+(E_x = 212 \text{ keV}) \rightarrow 3/2_{\text{g.s.}}^+$  transition is found to be considerably quenched;  $B(M1) = 1.0 \pm 0.1 \times 10^{-2} \mu_N^2$ , while that for the  $5/2_1^+(E_x = 333 \text{ keV}) \rightarrow 3/2_{\text{g.s.}}^+$  transition is normal;  $B(M1) = 8.2_{-1.8}^{+3.2} \times 10^{-2} \mu_N^2$  [8]. Conventional shell model Hamiltonians such as PSDMK2 and WBP [9] predict similar transition rates with normal order of magnitude for both transitions, as shown in Table I. The large quenching observed

TABLE I. Calculated and experimental  $B(M1)$  values in  $^{17}\text{C}$  for the transitions from the  $1/2_1^+$  and  $5/2_1^+$  states to the  $3/2_1^+$  ground state. The SFO, SFO' (no  $T = 1$  monopole corrections) and SFO-tls Hamiltonians, as well as PSDMK2 and WBP Hamiltonians are used for the shell model calculations. The numbers in parentheses [ ] are obtained with the effective  $g$  factors  $g_s^{\text{IV,eff}}/g_s^{\text{IV}} = 0.95$  and  $\delta g_\ell^{\text{IV}} = 0.15$ . See text for the loosely bound effects in the sixth row. Energies of the  $1/2_1^+$  and  $5/2_1^+$  states relative to the  $3/2_1^+$  state,  $E(1/2_1^+)$  and  $E(5/2_1^+)$ , are also shown.

Hamiltonian	$E(1/2_1^+)$ (MeV)	$B(M1; 1/2_1^+ \rightarrow 3/2_{\text{g.s.}}^+) (\mu_N^2)$	$E(5/2_1^+)$ (MeV)	$B(M1; 5/2_1^+ \rightarrow 3/2_{\text{g.s.}}^+) (\mu_N^2)$
SFO [4]	-0.304	0.0769 [0.0610]	0.341	0.0767 [0.0630]
PSDMK2 [12,13]	-0.305	0.0840	0.406	0.0695
WBP [9]	0.295	0.0780	0.032	0.0774
SFO'	-0.769	0.0667 [0.0530]	-0.005	0.1170 [0.0953]
SFO-tls	0.075	0.0320 [0.0228]	0.141	0.0856 [0.0674]
SFO-tls	0.032	0.0152 [0.0100]	0.064	0.0977 [0.0768]
+ loosely bound effects				
experiment [8]	0.212	$0.01 \pm 0.001$	0.333	$0.082 + 0.032/-0.018$

in the  $1/2_1^+ \rightarrow 3/2_{\text{g.s.}}^+$  transition is not reproduced, even in the case of the SFO Hamiltonian. The use of effective spin  $g$  factors,  $g_s^{\text{eff}}/g_s = 0.9 \sim 0.95$ , does not help to get sufficient reduction of the transition rate.

The ordering of calculated energy levels does not agree with the observed ones as shown in Fig. 2(a) and Table I for the PSDMK2 and SFO cases. For the WBP interaction,  $3/2_1^+$  is calculated to be the ground state which is consistent with experiment. The rms deviation of the energies obtained in the shell model calculations from the experimental values is about 150–300 keV. For example, it is 330 keV for the WBP Hamiltonian [9], and 150 (130) keV for the USD [18] (USDA [19]) Hamiltonian. Considering this deviation, the experimental energy levels of  $^{17}\text{C}$  are rather well reproduced by the WBP interaction.

We next show the results for  $M1$  transitions obtained by the shell model calculations with the present modified Hamiltonian, SFO-tls. The monopole corrections are made in the  $T = 1$  channel so that the ordering of the low-lying levels,  $3/2_1^+$ ,  $1/2_1^+$ , and  $5/2_1^+$ , becomes consistent with the observation. Namely, the monopole matrix elements for  $1s_{1/2}$ - $0d_{5/2}$  and  $0d_{3/2}$ - $0d_{5/2}$  orbits are made more repulsive by 0.58 MeV and 0.2 MeV, respectively, while those for

$0d_{5/2}$ - $0p_{1/2}$  and  $0d_{5/2}$ - $0p_{3/2}$  orbits are made more attractive by 0.1 MeV and 0.04 MeV, respectively. The attractive two-body matrix element for  $0d_{5/2}^2$  ( $J = 2$ ,  $T = 1$ ) is also enlarged by -0.2 MeV in order to enhance the quadrupole component of the interaction. More attraction between  $0d_{5/2}$  and  $0p$  orbits works to push down the neutron  $0d_{5/2}$  orbit, lowering the  $3/2_1^+$  and  $5/2_1^+$  states relative to the  $1/2_1^+$  state. The same effect is caused by lowering the  $0d_{5/2}^2$  ( $2^+$ ,  $T = 1$ ) state. Note that monopole matrix elements for  $1s_{1/2}$ - $0d_{5/2}$  orbits are more repulsive in both the USD [18] and SDPFM Hamiltonians [20] compared to the G-matrix elements [16]. Monopole matrix elements for  $0d_{3/2}$ - $0d_{5/2}$  orbits are made more repulsive in the SDPFM Hamiltonian [20] by 0.3 MeV compared to the USD Hamiltonian [18]. These repulsive monopole corrections in the  $T = 1$  channel are important to improve the evolutions of the  $sd$ -shell to get magic number  $N = 16$  toward drip-lines [21].

Results obtained with bare  $g$  factors and with effective isovector spin and orbital  $g$  factors,  $g_s^{\text{IV,eff}}/g_s^{\text{IV}} = 0.95$  and  $\delta g_\ell^{\text{IV}} = 0.15$ , are shown in Table I and Fig. 2(b). Calculated  $B(M1)$  values for the  $1/2_1^+ \rightarrow 3/2_{\text{g.s.}}^+$  transition for SFO-tls are rather close to the observed value. For the  $5/2_1^+ \rightarrow 3/2_{\text{g.s.}}^+$  transition, calculated  $B(M1)$  values are consistent with the

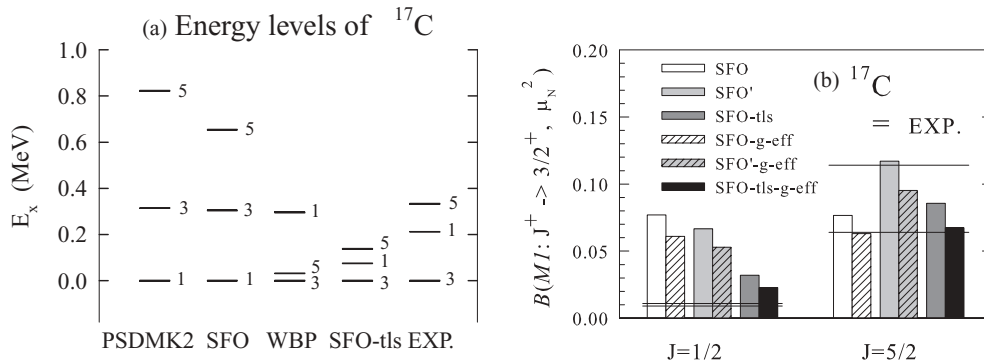


FIG. 2. (a) Energy levels of  $^{17}\text{C}$  obtained for PSDMK2, SFO, WBP, and SFO-tls Hamiltonians as well as experimental ones [8]. (b)  $B(M1)$  transition strengths for  $1/2_1^+ \rightarrow 3/2_{\text{g.s.}}^+$  and  $5/2_1^+ \rightarrow 3/2_{\text{g.s.}}^+$  transitions in  $^{17}\text{C}$ . Calculated values obtained by using SFO, SFO' and SFO-tls Hamiltonians are shown by white, gray, and dark-gray bars, respectively. SFO' denotes the case without the monopole corrections. Hatched and black bars denoted by  $g$ -eff are obtained with effective  $g$  factors (see text). Experimental values are taken from Ref. [8].

TABLE II. Calculated proton and neutron  $E2$  matrix elements and  $B(E2)$  values for the transitions to the  $2_1^+$  states of  $^{16}\text{C}$  and  $^{18}\text{C}$ . Results for the SFO, SFO-tls, WBP, and WBT Hamiltonians are given. Effective charges  $e_p = 1.16e$  ( $1.11e$ ) and  $e_n = 0.33e$  ( $0.27e$ ) are used for  $^{16}\text{C}$  ( $^{18}\text{C}$ ). Experimental  $B(E2)$  values [5,6] are also shown.

Hamiltonian	$A_p$ (fm)	$^{16}\text{C}$ $A_n$ (fm)	$B(E2)$ ( $e^2\text{fm}^4$ )	$A_p$ (fm)	$^{18}\text{C}$ $A_n$ (fm)	$B(E2)$ ( $e^2\text{fm}^4$ )
SFO [4]	1.20	8.27	17.0	1.58	9.87	19.5
SFO-tls	1.38	8.06	18.2	1.60	9.69	19.9
WBP [9]	1.39	8.32	19.0	1.67	9.74	20.1
WBT [9]	1.16	8.48	17.1	1.55	9.79	19.0
Experiment			$13.0 \pm 1.0 \pm 3.5$ [5] $20.8 \pm 3.7$ [6]			$21.5 \pm 1.0 \pm 5.0$ [5]

observed value. The observed anomalous difference between the  $B(M1)$  strengths of these two transitions is rather well explained by our new Hamiltonian, SFO-tls. Calculated energy levels are also consistent with observation as shown in Fig. 2(a).

We now discuss effects due to the enhancement of the tensor interaction.  $B(M1)$  values obtained without the monopole corrections in the  $T = 1$  channel are shown in Fig. 2(b) as SFO'. The tensor force induces some asymmetry between the two transitions, enhancing the  $B(M1; 5/2_1^+ \rightarrow 3/2_{\text{g.s.}}^+)$  value by about 50%, but reducing the  $B(M1; 1/2_1^+ \rightarrow 3/2_{\text{g.s.}}^+)$  value by only 13% (see Table I), which is not enough to reproduce the experimental value. The repulsive tensor force between the neutron  $0d_{5/2}$  and the proton  $0p_{3/2}$  orbit pushes up the neutron  $0d_{5/2}$  orbit, thus leading to less neutrons in  $0d_{5/2}$  shell but more neutrons in the  $1s_{1/2}$  shell, especially in the  $5/2_1^+$  state. The additional transition between the neutron  $1s_{1/2}$  orbits enhances the  $5/2_1^+ \rightarrow 3/2_{\text{g.s.}}^+$   $M1$  transition strength.

Further inclusion of the monopole corrections in the  $T = 1$  channel reduces the transition strengths in both  $M1$  transitions. Repulsion between the  $0d_{5/2}$  orbit and the  $1s_{1/2}$  or  $0d_{3/2}$  neutron orbits pushes down the neutron  $0d_{5/2}$  orbit, leading to more neutrons in the  $0d_{5/2}$  shell. The gap between the  $0d_{3/2}$  and  $0d_{5/2}$  orbits becomes larger, resulting in a quenching of the  $M1$  transition strengths. It can be shown that within the  $0d_{5/2}$ - $1s_{1/2}$  configuration space the  $M1$  strength vanishes exactly for the  $1/2_1^+ \rightarrow 3/2_{\text{g.s.}}^+$  transition. As three  $0d_{5/2}$  neutrons can couple only to  $3/2^+$ ,  $5/2^+$ , and  $9/2^+$ , the  $1/2^+$  state has a single configuration,  $|1/2^+\rangle = |0d_{5/2}^2(0^+) \times 1s_{1/2}\rangle$ . The  $3/2^+$  state is a linear combination of  $|0d_{5/2}^2(2^+) \times 1s_{1/2}\rangle$  and  $|0d_{5/2}^2(2^+, 4^+) \times 0d_{5/2}\rangle$  configurations. These states cannot be connected by the  $M1$  operator, that is,  $B(M1; 1/2^+ \rightarrow 3/2^+) = 0$ .

We now discuss the quadrupole ( $Q$ ) moment of  $^{17}\text{C}$ . A deformed Hartree-Fock (HF) calculation shows that  $^{17}\text{C}$  has a rather flat energy surface with two minima both at the prolate and oblate sides [22], which suggests a transitional nature of this nucleus. In fact, the  $Q$  moment of the  $3/2^+$  state, obtained by this HF calculation, is  $3.35\text{ fm}^2$  and  $-3.33\text{ fm}^2$  at the prolate and oblate minimum, respectively [22], while the difference between their energies is within about 1.2 MeV. Thus, substantial configuration and shape mixings are expected. On the other hand, in the shell model, the  $Q$  moment of

$^{17}\text{C}$  becomes as small as  $Q = 1.42\text{ fm}^2$  for SFO-tls, while the SFO gives  $Q = 2.82\text{ fm}^2$ . Proton and neutron effective charges are taken to be  $e_p = 1.2e$  and  $e_n = 0.3e$ . The reduction of the  $Q$  moment supports the transitional nature of the nucleus. It would be quite interesting to measure the  $Q$  moment of  $^{17}\text{C}$ .

We finally comment on  $E2$  transitions in the neighbor nuclei,  $^{16}\text{C}$  and  $^{18}\text{C}$ . The observed  $E2$  transition strengths are found to be hindered compared to systematics of empirical transition strengths [5,6]. The observed data are rather well explained by the use of the effective charges of Ref. [23], which decrease as the neutron number increases. Calculated  $B(E2)$  values as well as proton and neutron  $E2$  matrix elements for  $^{16}\text{C}$  and  $^{18}\text{C}$ , are shown in Table II. The proton  $E2$  matrix elements are small in  $^{16}\text{C}$  and  $^{18}\text{C}$ , and their values for SFO-tls or SFO are similar to those for WBP or WBT, which is consistent with the fact that the proton  $p$ -shell gaps are also similar (see Fig. 1). The matrix elements get larger for  $^{18}\text{C}$  than for  $^{16}\text{C}$ , since the proton  $p$ -shell gap decreases as the mass number increases from  $A = 16$  to  $A = 18$ , as shown in Fig. 1. The neutron  $E2$  matrix elements are large, and larger in  $^{18}\text{C}$  than in  $^{16}\text{C}$  because of the increase of the neutron excess. This increase of the neutron matrix elements is more or less compensated by the decrease of the neutron effective charge, thus resulting in similar contributions from neutrons in  $^{16}\text{C}$  and  $^{18}\text{C}$  within a few percents. The increase of the calculated total  $B(E2)$  values in  $^{18}\text{C}$ , compared to  $^{16}\text{C}$ , is about  $6 \sim 15\%$ , which can be attributed to the increase of the proton matrix elements. The calculated values are consistent with the experimental values [5,6], as shown in Table II. The gap between the  $0p_{1/2}$  and  $0p_{3/2}$  proton orbits is enhanced for  $^{16}\text{C}$  relative to  $^{18}\text{C}$ , in particular for the case of SFO-tls (see Fig. 1). The neutron effective charge induced by excitations of protons from the  $0p_{3/2}$  shell can thus change more in  $^{16}\text{C}$  than in  $^{18}\text{C}$ , leading to less neutron-excess dependence of the neutron effective charges and a steeper change of  $B(E2)$  values from  $^{16}\text{C}$  to  $^{18}\text{C}$ .

*Discussion and summary.* In summary, we have extended our work to improve shell model Hamiltonians [4,24] by taking fully into account the important role of the tensor force. Tensor and spin-orbit components of meson exchange forces have been used for the two-body matrix elements in the  $p$ - $sd$  crossing terms. With the fully revised  $T = 1$  monopole terms, our new Hamiltonian, SFO-tls, has been applied to study the structure of neutron-rich carbon isotopes. The anomalous



hindrance observed for the  $M1$  strength in the  $1/2_1^+ \rightarrow 3/2_{\text{g.s.}}^+$  transition in  $^{17}\text{C}$  is found to be rather well explained by SFO-tls, though there still remains a need for further quenching to reproduce the experimental value of the strength. Further quenching of the strength might be attributed to the loosely bound nature of neutrons in  $^{17}\text{C}$ . Low excitation energies of  $2^+$  states in  $^{18}\text{C}$  and  $^{20}\text{C}$  have been well explained by reducing the neutron-neutron interaction in the nucleus by 25% [25].

We wish to point out that the strength is particularly sensitive to a very few matrix elements involving the  $1s_{1/2}$  orbit, and further quenching can be obtained by reducing these matrix elements. For example, by reducing  $\langle 1s_{1/2}0d_{5/2}; J=2, T=1 | V | 0d_{5/2}0d_{3/2}; J=2, T=1 \rangle$  and  $\langle 1s_{1/2}0d_{5/2}; J=2, T=1 | V | 1s_{1/2}0d_{3/2}; J=2, T=1 \rangle$  by 20%, we obtain  $B(M1) = 0.0100(0.0152)\mu_N^2$  and  $B(M1) = 0.0768(0.0977)\mu_N^2$  with the effective (bare)  $g$  factors for the  $1/2_1^+ \rightarrow 3/2_{\text{g.s.}}^+$  and  $5/2_1^+ \rightarrow 3/2_{\text{g.s.}}^+$  transitions, respectively. These values are close to the observed values (see Table I). As the dominant contribution to the strengths comes from the neutron part of the transitions, these reductions of the  $T=1$  matrix elements lead to the same effect as a reduction of the neutron-neutron interaction.

The difference of the strengths of the two  $M1$  transitions, as well as the anomalous quenching of the  $M1$  transition strength from the  $1/2_1^+$  state, can also be obtained by starting from another Hamiltonian, where the  $sd$ -shell part is replaced by the SDPFM interaction [20]. Making the  $T=1$  monopole term for

$0d_{5/2}-1s_{1/2}$  orbits more repulsive by 0.3 MeV, we get  $B(M1) = 0.0360\mu_N^2$  and  $0.0872\mu_N^2$  for the transition from the  $1/2_1^+$  and  $5/2_1^+$  states, respectively, with effective isovector  $g$  factors of  $g_s^{\text{IV,eff}}/g_s^{\text{IV}} = 0.95$  and  $\delta g_\ell^{\text{IV}} = 0.15$ . A further quenching of the  $1/2_1^+ \rightarrow 3/2_{\text{g.s.}}^+$  transition strength is obtained, similarly to the SFO-tls case, if the loosely bound nature of neutron in  $^{17}\text{C}$  is taken into account. The calculated  $B(M1)$  values are  $0.0124\mu_N^2$  and  $0.1028\mu_N^2$  for the transition from the  $1/2_1^+$  and  $5/2_1^+$  states, respectively, which are close to the experimental values.

Thus, we find that exotic properties of  $M1$  transitions in  $^{17}\text{C}$  can be reproduced by improving shell model Hamiltonians, taking fully into account the tensor force as well as corrections in  $T=1$  monopole terms. A weaker neutron-neutron interaction is favored in order to reproduce the observed data. The origin of the repulsive corrections in  $T=1$  monopole terms may be attributed to three-body forces induced by  $\Delta$  excitations. Studies in this direction are now in progress.

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